# Single-Engine Failure after Takeoff: The Anatomy of a Turnback Maneuver 

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## Summary

In the latest Advisory Circular AC-83J, Appendix A.11.4 provides guidance to all CFI's relating to the subject: "Return to Field/Single-Engine Failure on Takeoff", i.e.,
(1) "Flight instructors should demonstrate and teach trainees when and how to make a safe 180-degree turnback to the field after an engine failure".
(2) "Flight instructors should also teach the typical altitude loss for the given make and model flown during a 180-degree turn, while also teaching the pilot how to make a safe, coordinated turn with a sufficient bank. These elements should give the pilot the ability to determine quickly whether a turnback will have a successful outcome. During the before-takeoff check, the expected loss of altitude in a turnback, plus a sufficient safety factor, should be briefed and related to the altitude at which this maneuver can be conducted safely. In addition, the effect of existing winds on the preferred direction and the viability of a turnback should be considered as part of the briefing".

The question that is most asked by Pilots about the turnback maneuver is "How high above the runway do I need to be before attempting a turnback maneuver?" In a recent NAFI webinar Capt Brian Schiff came up with a Rule-of-Thumb (ROT) regarding this altitude. Schiff's ROT is based on a 13-step flight experiment that the Pilot was required to perform. It required the Pilot to determine the altitude loss during a 45-degree banked gliding turn, at a speed no faster than best glide speed (or slightly lower), and after turning 360 degrees, the aircraft is rolled to a wings level attitude, and then flaring the aircraft to the point where the vertical speed went to zero. The total altitude loss during this flight experiment was designated as the "Observed Altitude Loss (OAL)". An additional $50 \%$ safety factor was then added to this altitude. Schiff designated this altitude as the "Turnback Height (TH)", Once these altitudes are determined, Schiff's ROT then comes into play, i.e.,

Do not consider a turnback maneuver unless:
(1) The aircraft has reached at least $2 / 3$ of the OAL when over the departure end of the runway (DER), and
(2) The aircraft has reached at least the TH

Unfortunately, the details of how this ROT was developed, and what flight scenarios were utilized, is not discussed. In addition, there is no information on how far from the DER the turnback maneuver was initiated. For example, if the Pilot reached the TH at $1 / 2$-mile from the DER, could one still make it back at $3 / 4$-mile from the DER? It is important to remember that every ROT must come with the limitations on when the ROT is to be used. Schiff does not provide this information.

As an Aerodynamicist for over five decades, one must question the validity of such a ROT. One would expect the altitude loss in the turnback maneuver to depend on the distance from the DER at which the turnback is initiated. However, if the aircraft was able to successfully execute a turnback maneuver at some distance from the DER, the magnitude of both the climb and glide angles would be a major factor in determining whether the aircraft could successfully execute a turnback maneuver at all distances beyond that point.

To satisfy the requirements of Appendix A11.4, we have developed a consistent steady-state aerodynamic analysis of the teardrop turnback maneuver under a no-wind condition. Although the no-wind condition is not the prevalent scenario for aircraft departures, it allows one to study both the exact geometry and the corresponding aerodynamics of the teardrop turnback maneuver. Understanding the geometry of the teardrop turnback maneuver provides a significant amount of information about the limitations of the turnback maneuver. We show that the teardrop turnback maneuver is composed of three segments. The first segment of the maneuver involves a gliding turn at some specified airspeed and bank angle, after which the Pilot rolls out on a heading that points the aircraft directly at the DER. In the second segment, the aircraft is in a wings-level glide at a specified speed. In the third segment, the aircraft is also in a gliding turn at a specified airspeed and bank-angle, which then, allows the aircraft to roll out over the runway centerline with just enough altitude remaining to flare the aircraft to a landing.

Knowledge of the geometry of the teardrop turnback maneuver under a no-wind condition provides the following important information:
(1) If the turnback maneuver is initiated at some distance $\bar{D}$ from the DER, after completion of the turn in Segment 1, the distance of the aircraft from the DER is exactly equal to $\bar{D}$.
(2) At the completion of Segment 1, the Pilot will have turned the aircraft $180+\Psi$ degrees, where $\Psi$ is intercept angle to the runway centerline. The angle $\Psi$ is only a function of $\frac{\bar{D}}{R_{1}}$, where $\mathrm{R}_{1}$, is the radius of the turn in Segment 1 , and depends on the aircraft TAS and bank angle flown. Thus, if the Pilot initiates the turnback at a distance $\bar{D}$ from the DER, the Pilot can only control the intercept angle by varying $\mathrm{R}_{1}$.
(3) During Segment 2, the aircraft is in a wings-level glide for a distance slightly less than $\bar{D}$.
(4) Segment 3 is a final gliding turn in which the Pilot turns the aircraft in the opposite direction $\Psi$ degrees, to roll out over the centerline of the runway. Thus, depending on the TAS and bank angle flown in Segment 3, there must be sufficient distance to lead the turn in order for the Pilot to roll out over the runway centerline, since overshooting the centerline could lead to the possibility of an accelerated stall.

Therefore, we see that during the turnback maneuver the aircraft will be turning a total of $180+2 \Psi$ degrees, plus gliding a distance slightly less than $\bar{D}$. This information allows one to make a quick estimate of the altitude loss in the turnback maneuver. However, limitations exist on how close to the DER the turnback maneuver can be initiated. If we constrain the maximum intercept angle to approximately 55 degrees, and also allow for a minimum bank angle of 15 degrees in Segment 3, we show that the turnback maneuver should not be initiated any closer than $\frac{\bar{D}}{R_{1}}=1.93$. Thus, just understanding the geometry of the teardrop turnback maneuver, provides us with a heads-up on some key limitations in performing the turnback maneuver.

It is important for all Pilots to understand that there are three key factors that determine the altitude loss in the teardrop turnback maneuver. These are:
(1) Aerodynamics of the specific aircraft being flown
(2) Environment: Windspeed and direction relative to the runway alignment
(3) Pilot skills in performing the maneuver

The aerodynamics is the key component because it determines the expected altitude loss (EAL) when performing the maneuver under a no-wind condition. The environment modifies the performance of the aircraft and thus the EAL. Finally, if the Pilot flies the aircraft according to the aerodynamics, then the outcome will be the EAL in the maneuver. However, non-optimal Pilot skills when executing this maneuver can only increase the EAL.

Since the altitude loss in the turnback maneuver is the sum of the altitude losses in all 3 segments, two of which are gliding turns, and one a wings-level glide, it is necessary to determine the altitude loss in both gliding turns and wings-level glides. An understanding of "Basic Aerodynamics" is all that the Pilot needs to determine this information. We define "Basic Aerodynamics" as the subject of Aerodynamics discussed in the latest Handbook of Aeronautical Knowledge (FAA 8083-25B, Chapter 5). It is based on the concept of steady-steady aerodynamics, which refers to the fact that one does not account for the time it takes to get from one aircraft attitude to the next. For example, if the bank angle went from zero to 45-degrees, the 45-degree bankangle would occur immediately. This approximation is routinely used in the field of

Aerodynamics and provides a reasonably accurate answer to the problem being analyzed.

With the knowledge of the geometry of the teardrop turnback maneuver, one can develop the aerodynamics to minimize the EAL in performing the turnback maneuver. For example, in Segment 2, our goal is to minimize the altitude loss per horizontal distance travelled. Thus, the Pilot needs to fly this segment at the angle-of-attack corresponding to the maximum L/D. Depending on the weight of the aircraft, the airspeed can be determined in flying this segment.

Segments 1 and 3 are gliding turns, and so the Pilot is trying to minimize the altitude loss per degree of turn. Again, "Basic Aerodynamics" provides the formula for the altitude loss per degree of turn $\left(\frac{d h}{d \theta}\right)$ as

$$
\frac{d h}{d \theta}=\left(\frac{\pi}{180}\right) \frac{F_{1} F_{3}}{F_{2} F_{4}}
$$

One determines this equation by dividing the rate of descent in feet/sec by the rate of turn in degrees $/ \mathrm{sec}$. Here, $\mathrm{F}_{1}, \mathrm{~F}_{2}, \mathrm{~F}_{3}$, and $\mathrm{F}_{4}$ are defined as:

$$
\begin{aligned}
& F_{1}=\left(\frac{4}{g}\right) \frac{W}{S} \\
& F_{2}=\rho \\
& F_{3}=\frac{1}{2 \sin \Phi \sqrt{\cos \Phi^{2}+(D / L)^{2}}} \\
& F_{4}=C_{L}\left(\frac{L}{D}\right)
\end{aligned}
$$

Where $\frac{W}{S}$ is the wing loading, g, the Earth's gravitational constant, $\rho$ the air density, $\Phi$ the bank angle, $C_{L}$ the lift coefficient, and $L / D$, the lift to drag ratio in the turn. Thus, all the information on how the Pilot should fly the gliding turn is in the above formula. It is easy to see the equation for $\frac{d h}{d \theta}$ contains the information about how aircraft weight, air density, bank angle, and angle-of-attack influence the altitude loss per degree of turn in a gliding turn. Here we identify one of the key parameters controlling the altitude loss per degree of turn as the ratio of the aircraft weight to the air density, i.e., $\frac{W}{\rho}$. Higher values of this ratio, i.e., gross weight and high-density altitudes, increase the OAL. Therefore "Performing a flight experiment in an attempt to determine the altitude loss per degree of turn under one set of conditions, cannot blindly be applied to a different set of conditions". Everything about how to scale the altitude loss per degree of turn is in the above equation.

In order to minimize the altitude loss in the turnback maneuver, we show that in the wings-level glide segment, the aircraft must be flown at the angle-of-attack for maximum L/D ratio, i.e. best glide speed. Whereas, in the gliding turn segments, the aircraft must be banked at 45 degrees, while flying just below the stall angle-of-attack, i.e. just above the accelerated stall speed for the weight of the aircraft.

Using the above information, we demonstrate how to create a chart which shows the EAL incurred during the turnback maneuver versus the distance $\bar{D}$, at which the turnback maneuver is initiated. This chart answers the key question, "How much altitude do we need for a "Potentially Successful Turnback Maneuver (PSTM)"? We use the term PSTM to indicate that both the aerodynamics and environment may allow for a PSTM, however, the non-optimal Piloting skills may not allow for a successful outcome.

However, the real question one should be asking is: "How can the Pilot utilize this information when the engine fails after departure? Although the Pilot may have this information in front of him/her, the Pilot must determine if the altitude the aircraft is at when the engine fails corresponds to the distance from the DER shown in the chart. This task is not something the Pilot should be engaged in during this emergency.

In order to mitigate the risk of having the correct altitude, but at the incorrect distance from the DER, we utilize an "inverse method", which uses the EAL in the turnback maneuver to obtain a "Required Minimum Runway Length (RMRL)" chart. This chart shows the minimum runway length as a function of distance from the DER, which would allow for a PSTM. The Pilot can review this chart prior to takeoff to determine when "Never to attempt a turnback maneuver". For example, if the runway length is greater than the RMRL, a PSTM can be anticipated. If the runway length is less than the RMRL, an "Impossible Turn" exists, and the Pilot should never attempt a turnback maneuver. Finally, the runway length may only allow for a PSTM over some range of distances from the DER. This can be a risky scenario for the turnback maneuver unless the Pilot is aware of this situation prior to departure.

We have chosen to utilize a C-172 as an example of how to generate the above chart and compare these results with the Schiff ROT for when to initiate a turnback maneuver. We show that the Schiff method does not adequately capture the true altitude loss in the turnback maneuver, since it does not consider the altitude loss in Segment 2. We have compared the aerodynamic model to the Schiff ROT for the case of a C-172 at gross weight, departing both a sea level airport (ISA), and a 5000-foot density altitude airport. In the case of the aircraft departing a sea level airport, we show that the Schiff ROT dismisses PSTM's between 800 and 2900 feet from the DER. This is due to the $50 \%$ safety factor on the "Observed Altitude Loss (OAL). In addition, Schiff's $2 / 3$ of the OAL requirement on the altitude above the DER is shown to be too low, since the aerodynamic model predicts $82 \%$ of the OAL would allow for a PSTM beyond 800 feet from the DER.

Schiff's ROT applied to the 5000-foot density altitude airport, shows PSTM's beyond 5100 feet from the DER. However, the aerodynamic model shows an "Impossible Turn" for all distances beyond 800 feet. One needs to increase the height over the DER to $95 \%$ of the OAL for a PSTM to occur between 800 and 4900 feet from the DER. Beyond 4900 feet from the DER, an "Impossible Turn" exists. This is due to the magnitude of the climb angle being less than the glide angle at this density altitude. Since the glide angle is independent of altitude, whenever a density altitude is reached for which the climb angle becomes less than the glide angle, such a situation is expected to occur. It is important that all Pilots are aware of this fact, since it confines a PSTM to a narrow region beyond the DER. These scenarios usually occur when flying aircraft with a relatively higher power loading, i.e., a C-172. Therefore, the author recommends extreme caution in initiating a turnback maneuver at a high-density altitude airport.

The consistent geometric/aerodynamic analysis developed here, can easily be used to develop the RMRL chart for the any aircraft being flown. In addition, it provides a method for optimizing the takeoff/climb profile to minimize the RMRL. For example, if the airspeed flown in Segment 1 for the initial turn is close to midway between $V_{x}$ and $\mathrm{V}_{Y}$, the aircraft will need to give up additional altitude in order to obtain its target $\mathrm{V}_{1}$ airspeed prior to executing the turnback maneuver. Thus, it may be better to fly an airspeed closer to $\mathrm{V}_{\mathrm{Y}}$ in the takeoff/climb profile to mitigate some of this loss of this altitude. In addition, we also show that the important parameter to monitor during the climb-out is the height of the aircraft passing over the DER, rather than a percentage of the OAL, since the RMRL is based on attaining this altitude over the DER.

Item (2) from Appendix A11.4 described in the beginning of the paper discusses the use of safety factors. Safety factors arise due to two uncertainties. First, the uncertainty in the aerodynamic data of the aircraft. These uncertainties can be quantified by flying the aircraft in both wings-level and gliding turns and backing out the aerodynamic data that is used in the analysis. The second uncertainty is due to the Pilot skills. It is important to understand that using large safety factors, such as in the Schiff ROT, can be overly conservative, in that it dismisses opportunities for PSTM's. When it comes to safety factors, it is important to take a "Surgical Approach". For example, in this aerodynamic model of the turnback maneuver, the altitude loss in each of the 3 segments is determined. In the "Surgical Approach", one can carefully apply different safety factors to each of the 3 segments, thereby reducing the overall conservativeness of the uncertainty.

Although the aerodynamic model of the turnback maneuver can determine the exact ground track of the aircraft under any wind condition, we provide a simple method to account for the wind effect on the turnback maneuver. In this method, the climb angle and glide path angle are corrected for the wind and are then used to determine the altitude loss in the turnback maneuver, which in turn, is then used to develop the RMRL chart.

We should point out that all the results shown in the paper were developed in a simple Excel Workbook, making it very easy for every Pilot to develop and display the charts needed to fully understand the effect of varying the key parameters on the potential success of executing a turnback maneuver. The Excel Workbook can be loaded onto an IPAD or other mobile device and used in the preflight preparation for the flight. However, the key takeaway is that the method used in the paper allows the Pilot to make the decision on the ground, rather than after departing the airport.

The author recommends a standardized training program for those Pilots interested in becoming proficient in executing the turnback maneuver. This program should include at least the following subjects:
(1) Aerodynamics of the turnback maneuver
(2) Stick and rudder skills in performing the turnback maneuver
(3) ADM and risk management in the decision to execute the turnback maneuver

Finally, it is important that all Pilots understand that acquiring the proper knowledge of the aerodynamics of the turnback maneuver is just as important as possessing the stick and rudder skills necessary to perform the turnback maneuver. We should all remember the adage, "The Devil is in the Details". If we do not heed this adage, all we can look forward to, is becoming just another NTSB accident statistic.

## 1. Introduction

In a recent webinar by Capt. Brian Schiff, the speaker discussed how the socalled "Impossible Turn" could become the "Possible Turn" when discussing the engine failure after takeoff in a single-engine aircraft. His goal was to develop a simple worksheet to determine the so-called "Minimum Turnback Altitude" that would be required for a PSTM. The turnback maneuver was based on a teardrop procedure for returning to the runway. Schiff describes the experiment as follows:
(1) On a cardinal heading, establish a stabilized climb halfway between $V_{X}$ and $V_{Y}$.
(2) Upon reaching a safe cardinal altitude, retard the throttle.
(3) Do nothing for 5 seconds and hold the nose up without stalling.
(4) After 5 seconds, simultaneously roll the aircraft into a 45-degree banked turn and pitch for no faster than best glide speed (or slightly lower).
(5) Continue this maneuver until completing a 360-degree turn.
(6) Roll out of the turn.
(7) Perform a moderately aggressive flare to simulate a landing.
(8) Note the altitude when the vertical speed becomes zero.
(9) The resultant altitude loss during the gliding turn is termed the" Observed Altitude Loss (OAL)".
(10) Increase the OAL by $50 \%$ to arrive at the altitude lost in the teardrop turnback maneuver, or as Schiff calls it, the "Turnback Height (TH)".
(11) Add this altitude lost to the airport elevation to determine the "Minimum Turnback Altitude".

Schiff also indicates that this procedure is for a "given aircraft and configuration". I assume configuration would mean aircraft weight, flap setting, etc. Since Schiff puts forth a ROT for this "turnback altitude", one would expect to see the limitations under which this ROT can be used. Schiff does not provide any. In addition, there is no process described that shows how this ROT was developed and validated.

After digesting the above procedure, one must ask the following questions:
(1) How does one scale the altitude lost in the Schiff experiment to a different aircraft weight configuration?
(2) How does one scale the altitude lost in the Schiff experiment to the higher density altitude airports
(3) Does it make sense that this so-called "Turnback Altitude" would be independent of the distance from the departure end of the runway where the turnback maneuver is initiated? For example, if the "Turnback Height" is 800 feet, will that altitude allow the aircraft to make it back to the runway whether the turnback maneuver is initiated at a point $1 / 2$ mile or $3 / 4$ mile from the departure end of the runway.

To answer these questions, we need to take a step back and view the teardrop turnback maneuver through a different lens. One can simplify the teardrop turnback maneuver in a way that the results provide a significant amount of insight into a real scenario of a turnback maneuver. The simplification is that we assume the windspeed is zero. Although this is not the most common scenario experienced during a takeoff, it does occur in the realm of flight. In fact, it can be shown to be a conservative estimate for the expected altitude lost (EAL) in the turnback maneuver. It also relates to Murphy's Law as it pertains to the turnback maneuver. Here Murphy's Law states "After departing into the wind, as soon as the engine fails, the windspeed goes to zero".

The analysis that is developed in this paper is based on a formal 3-step approach. In Step 1, we analyze the geometry of the teardrop turnback maneuver. Understanding the geometry of the maneuver, including the geometric properties of the teardrop maneuver, provides us with a significant amount of information about the limitations of the maneuver. In Step 2, we use this information to develop a simple, but accurate steady-state aerodynamic model of the teardrop turnback maneuver. The aerodynamic model provides all the necessary information on how to fly the maneuver. The results of the aerodynamic model allow us to determine the EAL in the turnback maneuver as a function of distance from the departure end of the runway (DER). Finally, in Step 3, we specify a takeoff/climb profile for the aircraft, and using the information in Step 2, we create a chart which determines the "Required Minimum Runway Length (RMRL) as a function of distance from the DER, that will allow for a PSTM. This chart can be reviewed prior to departure, and based on the aircraft aerodynamics, will let the Pilot know when "Never to Attempt a Turnback Maneuver".

In Section 2, we study the geometry of the teardrop turnback maneuver, to understand the important limitations when executing this maneuver. In Section 3, we develop the aerodynamic model of the turnback maneuver. In Section 4, we develop the RMRL chart for a C-172. In Section 5, we compare the aerodynamic model of the turnback maneuver with the Schiff Rule-of-Thumb, for determining when to initiate a turnback maneuver. In Section 6, we summarize the conclusions of the White Paper. Finally, in Appendix A, we develop a simple conservative wind correction algorithm to account for the effect of the wind on the RMRL.

## 2. Teardrop Turnback Maneuver Geometry

We will start by just considering the geometry of the teardrop turnback maneuver. We see that the aircraft climbs out along the extended centerline of the runway and at some distance from the departure end of the runway (DER), the engine fails. Whether you consider the lag time of 5 seconds or not does not make a difference in understanding the geometry of the turnback maneuver. Figure 1 show the geometry of the teardrop turnback maneuver. If at some distance $\overline{\mathrm{D}}$, from the DER, the aircraft initiates a gliding turn using some undetermined bank angle and airspeed, then rolls out on a heading that points the nose of the aircraft directly at the DER, the distance from that point to the DER is exactly equal the distance $\overline{\mathrm{D}}$ along the extended centerline of the runway. This is a geometric property of the teardrop maneuver.


Figure 1: Geometry of the Teardrop Turnback Maneuver.

The teardrop turnback maneuver is composed of three segments. Segment 1 is a gliding turn initiated from point $\overline{\mathrm{D}}$ on the extended centerline of the runway. This segment is flown at some predetermined bank angle and airspeed (i.e. $\Phi_{1}$ and $\mathrm{V}_{1}$ ). At the end of this segment, the aircraft rolls out on a heading that points the nose of the aircraft directly at the DER. The angle $\Psi$ is the intercept angle to the runway centerline. For a teardrop geometry, the intercept angle is only a function of the parameter $\frac{\overline{\mathrm{D}}}{R_{1}}$,
where $R_{1}$ is the radius of the turn in segment 1 . Figure 2 shows the intercept angle versus $\frac{\overline{\mathrm{D}}}{R_{1}}$. This figure provides considerable insight into the observed variation. For example, for a given value of $R_{1}$, as $\bar{D}$ gets very large, we see that the intercept angle starts to approach zero. Consider making a 180 -degree turn two miles from the DER. At this distance, if $\mathrm{R}_{1}$ is 400 feet, $\bar{D} / \mathrm{R}_{1}=26.4$. Therefore, to the Pilot, it appears that the aircraft is close to the centerline of the runway at the completion of the 180-degree turn. Thus, in this scenario, the total number of degrees turned in segment 1 approaches 180 degrees (i.e. $\Psi=0$ ).


Figure 2: Variation of the Intercept angle versus $D / R_{1}$

Now, consider another limiting case. In this scenario, the aircraft initiates the turnback maneuver at a distance $\bar{D}=\mathrm{R}_{1}$. Figure 3 shows this scenario, sometimes termed the "270-90 Turnback Maneuver". Since $\bar{D} / R_{1}=1$, the intercept angle is 90 degrees. In this scenario the aircraft turns 270 degrees from the extended centerline of the runway and approaches perpendicular to the DER. This can be a dangerous scenario, since the aircraft must then make a 90 -degree turn in the opposite direction to the turn in Segment 1 at low altitude to align the aircraft with the runway centerline. In the case of a wind, this scenario sets the Pilot up for an overshoot of the runway centerline, leading to the possibility of a fatal accelerated stall. In the "270-90" scenario, the aircraft is required to turn a total of 360 degrees. One observation of this scenario is initiating a turnback maneuver too close to the DER can easily lead to an "Impossible Turn".

## 270-90 Turn-back Scenario



Figure 3: "270-90" Turnback Scenario

Continuing on to segment 2, we observe that in this segment the aircraft is gliding with the wings level at a predetermined speed $\mathrm{V}_{2}$, for a distance somewhat less than $\bar{D}$, after which, the aircraft enters Segment 3, a final turning segment at a predetermined bank angle and airspeed (i.e. $\Phi_{3}, V_{3}$ ). Specifying both $\Phi_{3}$ and $V_{3}$ will determine the radius of the turn in segment 3 , i.e. $\mathrm{R}_{3}$.

In Segment 3, the aircraft will be turning somewhere between 0 and 90 degrees, depending on the ratio of $\bar{D} / R_{1}$. It is best to avoid a large intercept angle in Segment 3, since large angles may not provide sufficient lead for the final turn so that the aircraft will overshoot the runway centerline after rolling out on the runway heading. Thus, one can set a limit on the maximum value of the intercept angle. Note that a 55-degree intercept angle will occur when $\frac{\bar{D}}{R_{1}}=1.92$. In addition, it can also be shown that in order to ensure a sufficient lead for the final turn when using a minimum bank angle of 15 degrees, no turnback maneuver should be initiated less than $\frac{\bar{D}}{R_{1}}=1.93$ from the DER.

Thus, avoiding a turnback maneuver when $\frac{\bar{D}}{R_{1}} \leq 2$ would keep the Pilot out of a potential "Impossible Turn" situation. We will discuss this issue later in the paper when we determine the bank angles and airspeeds for the 3 segments of the turnback maneuver.

With the above understanding of the geometry of the teardrop turnback maneuver, we can now develop an aerodynamic model of the turnback maneuver. In Step 2, we demonstrate how one determines the formulas necessary to obtain the expected altitude loss (EAL) for both wings level glides and gliding turns.

## 3. Aerodynamic Analysis for Determining the Altitude Loss during both Wings Level Glides and Gliding Turns

This task requires an understanding of "Basic Aerodynamics". I define "Basic Aerodynamics" as the subject of Aerodynamics discussed in the latest Handbook of Aeronautical Knowledge (FAA 8083-25B, Chapter 5).

We will start with Segment 2, the wings level glide portion of the turnback maneuver. The first question one asks is "What is the Pilot attempting to accomplish in this segment?". Clearly, the goal is to lose the minimum amount of altitude over the distance $\bar{D}$. To accomplish this goal, the Pilot must fly the aircraft at the angle-of-attack for maximum L/D. From an Aerodynamicist standpoint, it is best to use angle-of-attack rather than airspeed, since the airspeed for best glide is a function of the weight of the aircraft. However, one should know the maximum L/D occurs at a fixed pitch attitude, independent of both the weight of the aircraft and the altitude. One can derive this result by noting that the maximum L/D occurs at given angle-of-attack, and a given L/D results in given flight path angle. Since the pitch attitude is related to both the angle-of-attack and the flight path angle, the pitch attitude is constant, and is independent of the aircraft weight and altitude.

To determine the altitude loss in this segment, one can go the Emergency Section of the POH and look for the "Maximum Glide" figure. For example, for a C-172, the L/D is approximately 9.1. Therefore, for every thousand feet traveled horizontally, the aircraft will lose approximately 110 feet of altitude. Thus, we now know how to determine the altitude loss in Segment 2. In addition, we also know the aircraft will be flown at an airspeed corresponding to maximum L/D (i.e. best glide speed) for the corresponding weight of the aircraft. In the case of a C-172 at gross weight, the best glide speed is 65 KCAS.

We now turn our attention to the aerodynamics of the turning segments of the turnback maneuver. We start with Segment 1. From Figure 1, we see that we need to determine the bank angle and angle-of-attack to fly this segment. Since the aircraft is in a gliding turn in this segment, the goal here is to minimize the altitude loss per degree of turn (i.e., $\frac{d h}{d \theta}$ ). Recall, the total number of degrees of turn required is 180 degrees plus the intercept angle, i.e., $\theta_{\text {Total }}=180+\Psi$. So whatever number of degrees one needs to turn, the Pilot needs to minimize the altitude loss per degree of turn.

To determine the altitude loss per degree of turn, we need to divide the rate of descent of the aircraft by the rate of turn of the aircraft. The formulas for both quantities are given in FAA 8083-25B. We can express the rate of descent of the aircraft in $\mathrm{ft} / \mathrm{sec}$ as

$$
\begin{equation*}
\frac{d h}{d t}=V_{T A S} \operatorname{Sin} \gamma_{g} \tag{1}
\end{equation*}
$$

Here $\gamma_{g}$ is the glide angle in degrees, and $V_{\text {TAS }}$ is the aircraft TAS in feet per second. Similarly, the aircraft rate of turn in radians/sec is given by

$$
\begin{equation*}
\frac{d \theta}{d t}=\frac{V_{T A S}}{R} \tag{2}
\end{equation*}
$$

Here, R is the radius of the turn in feet. Using the turn dynamics equation, we can express $R$ as

$$
\begin{equation*}
R=\frac{V_{T A S}^{2}}{g \operatorname{Tan} \Phi \operatorname{Cos} \gamma_{g}} \tag{3}
\end{equation*}
$$

Where $\Phi$ is the bank angle, and $g$ is the Earth's gravitational constant (i.e.
$32.174 \mathrm{ft} / \mathrm{sec}^{2}$ ).
Using these formulas, we can express $\frac{d h}{d \theta}$ as follows:

$$
\begin{equation*}
\frac{d h}{d \theta}=R \operatorname{Sin} \gamma_{g} \tag{4}
\end{equation*}
$$

After a little algebra, the resulting expression for the altitude loss per degree of turn is given by

$$
\begin{equation*}
\frac{d h}{d \theta}=\left(\frac{\pi}{180}\right) \frac{F_{1} F_{3}}{F_{2} F_{4}} \tag{5}
\end{equation*}
$$

We have grouped the parameters in the above equation in such a manner that they are essentially independent of each other. We define the $F_{i}$ 's as follows:
(a) $F_{1}=\left(\frac{4}{g}\right) \frac{W}{S}$ is the aircraft wing loading function, where g is the Earth's gravitational acceleration, $W$ the aircraft weight, and $S$ the wing area in $\mathrm{ft}^{2}$
(b) $F_{2}=\rho$ is the air density, units are in slugs $/ \mathrm{ft}^{3}$ (i.e., sea-level density is 0.002378 slugs $/ \mathrm{ft}^{3}$ )
(c) $F_{3}=\frac{1}{2 \sin \Phi \sqrt{\cos \Phi^{2}+(D / L)^{2}}}$ is defined as the "Bank Angle Function", with $\Phi$ the bank angle, and $D / L$ is the inverse of the lift to drag ratio flown in the turn
(d) $F_{4}=C_{L}\left(\frac{L}{D}\right)$ is defined as the "Aerodynamic Function", and only depends on angle-of-attack

We should point out that while $F_{1}, F_{2}$, and $F_{4}$ are completely independent of each other, i.e., changing the value of one of the $F_{i}$ ' $s$ does not change any of the others, $F_{3}$ has a very weak coupling to the aerodynamics of the aircraft through the L/D ratio. Thus, changing $\mathrm{F}_{4}$, would have a very slight effect on the change in $\mathrm{F}_{3}$.

We now have all the information we need to determine the correct bank angle and angle-of-attack to fly in Segment 1 . To minimize the altitude loss per degree of turn, we need to minimize $F_{1}$ and $F_{3}$, and maximize $F_{2}$ and $F_{4}$. However, $F_{1}$ and $F_{2}$ are not minimized since they define the conditions under which the aircraft is being flown. However, it does show both higher aircraft weight and density altitude will lead to increased altitude loss per degree of turn. Since $\mathrm{F}_{1}$ is proportional to the aircraft weight, performing the results of the Schiff experiment at one particular weight will need to be scaled if the results of the experiment were to be used with a different aircraft weight at takeoff. Thus, if one performed the Schiff experiment with the Pilot and CFI aboard the aircraft (i.e., $10 \%$ below gross weight) and determined the "Observed Altitude Loss (OAL)", there would be a $10 \%$ increase in the OAL if the aircraft encountered the engine failure at gross weight at the same altitude. We have now answered question (1) in the beginning of this paper. Figure 4 shows the correction for the weight when performing the experiment at a weight other than the gross weight of the aircraft. Note that this figure is predicated on flying the aircraft at the airspeed based on the reduced weight of the aircraft.

## Weight Effect on Altitude Loss in Turn-back Maneuver ( $\mathrm{F}_{1}$ )

| Percent Below Gross Weight | Percent Reduction in Altitude Loss |
| :---: | :---: |
| 0 | 0 |
| 5 | 5 |
| 10 | 10 |
| 15 | 15 |
| 20 | 20 |



Predicated on appropriate reduction of airspeeds with reduction in weight

Figure 4: Weight Effect on Altitude Loss in Turnback Maneuver ( $\mathrm{F}_{1}$ )

Next, let us now assume the Schiff experiment was performed at an altitude of 3000 MSL. If the same weight of the aircraft was used to perform the experiment at a density altitude of 6000 feet, the OAL would be approximately $20 \%$ more than that recorded in the original experiment. This variation is shown in Figure 5. Note, that Figures 4 and 5 reference the changes in the altitude loss per degree of turn to gross weight and sea level altitude. Thus, one should perform the experiment using a gross weight and sea level altitude, and then scale the results using Figures 4 and 5. Under these conditions

$$
\begin{equation*}
\left(\frac{F_{1}}{F_{2}}\right)=\left(\frac{4}{g}\right) \frac{W_{G W}}{S \rho_{S L}} \tag{6}
\end{equation*}
$$

Consider that we set up the Schiff experiment such that

$$
\begin{equation*}
\left(\frac{4}{g}\right) \frac{W}{S \rho}=\left(\frac{4}{g}\right) \frac{W_{G W}}{S \rho_{S L}} \tag{7}
\end{equation*}
$$

This would result in the following formula

$$
\begin{equation*}
\frac{W}{W_{G W}}=\frac{\rho}{\rho_{S L}} \tag{8}
\end{equation*}
$$

Thus, if one chose the altitude for the Schiff experiment, the above formula would tell the Pilot how much the aircraft should weigh in order for the altitude loss per degree of turn be equivalent to the turnback maneuver being performed at gross weight at a sea level airport. However, the Pilot will need to fly the same bank angle at the appropriate lower speed that corresponds to this reduced weight to be at the same angle-of-attack. As an example, consider the gross weight of the aircraft to be 2300 lbs . If the Schiff experiment is performed at an altitude of 3000 MSL (standard atmosphere), the aircraft should weigh approximately $9 \%$ below gross weight in order to obtain the altitude loss per degree of turn corresponding to the aircraft at gross weight at sea level. We have now answered question (2) in the beginning of this paper. Recall that the Schiff rule-of-thumb adds a 50\% safety factor to the OAL for the 360-degree turn, but does not correct for the weight and altitude difference between the experiment and the actual conditions under which the turnback maneuver is flown. This is clearly a deficiency in the Schiff model, since it eats into the $50 \%$ safety factor that Schiff adds to OAL.

## Density Altitude Effect on Altitude Loss

in the Turn-back Maneuver $\left(\mathrm{F}_{2}\right)$

| Density Altitude (feet) | Percent Increase in Altitude Loss |
| :---: | :---: |
| Sea Level | 0 |
| 1000 | 3 |
| 2000 | 6 |
| 3000 | 9 |
| 4000 | 13 |
| 5000 | 16 |
| 6000 | 20 |
| 7000 | 23 |
| 8000 | 27 |

Figure 5: Density Correction for $\mathrm{F}_{2}$ Referenced to Sea Level Density

The remaining two parameters, $\mathrm{F}_{3}$ and $\mathrm{F}_{4}$, are now used to determine the optimum bank angle and angle-of-attack while flying the aircraft in Segment 1. Since we need to minimize the Bank Angle Function, we determine that this will occur when the bank angle is given by

$$
\begin{equation*}
\Phi=\frac{1}{2} \operatorname{Cos}^{-1}\left[-\left(\frac{D}{L}\right)^{2}\right] \tag{9}
\end{equation*}
$$

Here, $\frac{D}{L}$ is the inverse of the lift to drag ratio flown in the turn. Figure 6 shows a plot of the Fs function versus bank angle using the L/D ratio used to fly the aircraft in Segment 1. In the case of a C-172, F3 minimizes at a bank angle of 45.4 degrees. However, notice how flat the curve is at the minimum value of $\mathrm{F}_{3}$. The Pilot could fly the turnback maneuver anywhere between 40 and 50 degrees without a significant impact on the altitude loss per degree of turn.


Figure 6: F3 Function versus Bank Angle

Finally, directing our attention to the $\mathrm{F}_{4}$ function. Here one needs to maximize the Aerodynamic Function. This requires maximizing the quantity $C_{L}\left(\frac{L}{D}\right)$. We can rewrite $\mathrm{F}_{4}$ as

$$
\begin{equation*}
F_{4}=C_{L}\left(\frac{L}{D}\right)=C_{L}\left(\frac{C_{L}}{C_{D}}\right)=\frac{C_{L}^{2}}{C_{D}} \tag{10}
\end{equation*}
$$

However, using the drag polar of the aircraft, i.e. expressing the drag coefficient as the sum of the parasite and induced drag, and expressing the induced drag in terms of the lift coefficient, we obtain the following formula for the drag polar

$$
\begin{equation*}
C_{D}=C_{D_{0}}+\kappa C_{L}^{2} \tag{11}
\end{equation*}
$$

Here, $C_{D_{0}}$ is the parasite drag coefficient, and ${ }_{\kappa}$ is a coefficient that depends on both the planform shape of the wing and the wing aspect ratio. We can now express $\mathrm{F}_{4}$ as

$$
\begin{equation*}
F_{4}=\frac{C_{L}^{2}}{C_{D_{0}}+\kappa C_{L}^{2}} \tag{12}
\end{equation*}
$$

Figure 7, taken from the Handbook of Aeronautical Knowledge (FAA 8083-25B), shows the behavior of $L / D$ ratio and both the lift and drag coefficients corresponding to a generic aircraft.


Figure 7: Aerodynamic Coefficients and L/D Ratio for Generic Aircraft

Note, at small angles-of-attack, the drag coefficient approaches a constant value. This limit is the parasite drag coefficient $C_{D_{0}}$. If we pick any other angle-of attack, using the lift and drag coefficients, we can then determine the value ${ }_{\kappa}$. Thus, using Figure 7, the function $\mathrm{F}_{4}$ can be created for any aircraft. However, if we assume that the Pilot does not have Figure 7 available for the aircraft being flown, the $\mathrm{F}_{4}$ function can be created using a somewhat different approach. One can determine the coefficients in the above formula by using the glide chart in the POH . When the aircraft is flying at the maximum $L / D$, the induced drag is equal to the parasite drag. Since the L/D ratio can be determined by calculating the slope of the altitude loss versus distance curve in the POH glide chart, one can show the parasite drag coefficient is given by

$$
\begin{equation*}
C_{D_{0}}=\frac{\left(C_{L}\right)_{\max L / D}}{2\left(\frac{L}{D}\right)_{\max }} \tag{13}
\end{equation*}
$$

To determine the value of $C_{D_{0}}$ we need the lift coefficient at maximum L/D. Since the glide chart provides an airspeed to fly for a given weight of the aircraft (i.e. usually gross weight), we can calculate the required lift coefficient as

$$
\begin{equation*}
\left(C_{L}\right)_{\text {MaxLID }}=\frac{(W / S) \cos \gamma_{b g}}{\frac{1}{2} \rho_{S L} V_{b s}{ }^{2}} \tag{14}
\end{equation*}
$$

Here $V_{b g}$ is the best glide calibrated airspeed, $\rho_{s L}$ is the density at sea level, and $\gamma_{b g}$ is the glide angle at $\max \mathrm{L} / \mathrm{D}$ and is given by $\tan \gamma_{b_{g}}=\frac{1}{(L / D)_{\text {max }}}$. The remaining parameter $\kappa$, can be determined using the formula

$$
\begin{equation*}
\kappa=\frac{1}{2\left(\frac{L}{D}\right)_{\max }\left(C_{L}\right)_{\max L / D}} \tag{15}
\end{equation*}
$$

At this point, the $\mathrm{F}_{4}$ is determined for any aircraft. Since both the value of $C_{D_{0}}$ and ${ }_{\kappa}$ are positive numbers, it is easy to show that the $\mathrm{F}_{4}$ will be a maximum at the maximum value of the lift coefficient, i.e. $C_{L_{\max }}$. This will occur just as we approach the stall angle-of-attack. Since the aircraft will be in a gliding turn, the accelerated stall speed of the aircraft is given by

$$
\begin{equation*}
V_{S_{a c}}=\sqrt{n} V_{S_{1 g}} \tag{16}
\end{equation*}
$$

where n is the load factor during the turn, and $V_{S_{1 g}}$ is the $1-\mathrm{g}$ stall speed of the aircraft. Since the optimum bank angle in Segment 1 is 45 degrees, and assuming the shallow flight path approximation (i.e., $\gamma_{g} \leq 12 \mathrm{deg}$ ), the load factor on the aircraft would be approximately 1.41 . Therefore, the correct speed to fly in Segment 1 is not the speed for maximum L/D (i.e. maximum glide), but the speed corresponding to the angle-of-attack for maximum CL. Clearly, it is not recommended to fly the gliding turn at just above the accelerated speed, and one would need to add about $10 \%$ pad to the accelerated stall speed when flying this gliding turn. Figure 8 shows the $\mathrm{F}_{4}$ function versus the lift coefficient for a C-172. As can be seen in the Figure 8, there is approximately a 10\% impact in the altitude loss per degree of turn when adding the $10 \%$ airspeed pad.


Figure 8: F4 Function versus Lift Coefficient

We now turn our attention to Segment 3 of the turnback maneuver. Since the final segment is a turning segment, the aerodynamics are identical to that discussed for Segment 1. The final turning angle, $\Psi$, is somewhere between 0 and 90 degrees. However, since Segment 3 is being flown close to the ground, with a potential for overshooting the runway centerline, one needs to view this final segment with caution. Since the aircraft is transitioning from a wings level gliding flight in Segment 2, to a gliding turn in Segment 3 at low altitude, rolling into a steep bank at that point leaves too much of a chance for the Pilot to lose precious remaining altitude during the bank. If we initiate the bank angle at 15 degrees at the same airspeed flown in the Segment 2, increasing the bank gradually as necessary in order to avoid a runway overshoot, the Pilot can avoid an accelerated stall by keeping the maximum bank angle to no more than 45 degrees. Recall, the actual distance flown in Segment 2 is somewhat less than $\bar{D}$, since the final turn needs to be initiated prior to the DER. Note that the F ${ }_{3}$ function in Figure 6 is about a factor of 2 higher with a 15 -degree bank, than at a 45 -degree bank. Thus, holding a 15-degree bank angle will give rise to a larger altitude loss in Segment 3 than with a 45-degree bank angle. However, this is accounted for in the aerodynamics.

The lead for initiating the final turn, i.e., the distance from the DER to the beginning of Segment 3, as shown in Figure 1, is given by

$$
\begin{equation*}
L_{0}=R_{3} \operatorname{Tan}\left(\frac{\psi}{2}\right) \tag{17}
\end{equation*}
$$

Here $R_{3}$ is the radius of the turn in Segment 3, and $\Psi$ is the previously determined intercept angle. The radius $\mathrm{R}_{3}$ is determined by the true airspeed and bank angle flown in Segment 3. In addition, it is important for the Pilot to understand that the final turn will end over the centerline of the runway and the aircraft will run out of altitude at some distance down from the DER. This distance down the runway from the DER where the altitude above the runway is zero, is also shown in Figure 1, and is given by

$$
\begin{equation*}
L_{D}=R_{3} \operatorname{Tan}\left(\frac{\psi}{2}\right) \tag{18}
\end{equation*}
$$

Figure 9 show both $L_{d}$ and $L_{0}$ as a function of the intercept angle. Note, under no wind conditions, we see that $L_{0}=L_{D}$.


Figure 9: Segment 3 Lead Distance, Lo and Unusable Runway Length, Lo versus $\Psi$

To obtain the altitude loss in the turnback maneuver, we add the altitude loss in the 3 segments together. Tables 1 and 2 show the $\mathrm{C}-172$ aerodynamic parameters used to determine the altitude loss in the 3 segments. The aircraft is at gross weight (2300 lbs.) and the turnback maneuver is being flown at sea level under no wind conditions.

Table 1: C-172 Parameters Used to Determine Altitude Loss in Turnback Maneuver

| Parameter | Value |
| :---: | :---: |
| $V_{S}=$ Clean Stall Speed 1-G | 50 KCAS |
| W/S $=$ Wing Loading at Gross Weight | $13.2(\mathrm{lb} . / \mathrm{ft} \wedge 2)$ |
| $C_{L_{\max }}=$ Maximum Lift Coefficient | 1.556 |
| $\mathrm{~V}_{\mathrm{bg}}=$ Best glide speed | 65 KCAS |
| $(\mathrm{L} / \mathrm{D})_{\max }=$ Maximum Lift to Drag Ratio | 9.09 |
| $C_{D_{0}}=$ Parasite Drag Coefficient | 0.0506 (propeller windmilling) |
| $\mathrm{k}=$ Induced Drag Coefficient | 0.0339 (engine idling) |
| $\left(C_{L} \frac{L}{D}\right)_{\max }$ | 0.0597 |

Table 2: C-172 Parameters Used in the 3 Segments of Turnback Maneuver

| Parameter | Segment 1 | Segment 2 | Segment 3 |
| :---: | :---: | :---: | :---: |
| $\mathrm{V}(\mathrm{KCAS})$ | 65 | 65 | 65 |
| $\phi(\mathrm{deg})$ | 45 | 0 | 15 |
| $C_{L}$ | 1.304 | 0.9224 | 0.9549 |
| $C_{D}$ | 0.1522 | 0.1014 | 0.1050 |
| $L / D$ | 8.57 | 9.09 | 9.09 |
| $C_{L}(L / D)$ | 11.17 | 8.39 | 8.68 |
| $\gamma_{\text {glide }}($ deg $)$ | 9.37 | 6.28 | 6.50 |
| Rate of Descent (fpm) | 1072 | 720 | 745 |
| Radius of Turn $(\mathrm{ft})$ | 379 | Wings Level | 1.0 |
| Load Factor | 1.414 | $110 \mathrm{ft} / 1000 \mathrm{ft}$ horizontal | 2.79 |
| $(d h / d \theta)(\mathrm{tt} / \mathrm{deg})$ | 1.08 |  | 1.035 |

Note that we chose to keep the airspeed in Segment 2 and 3 the same, however, in the case of a C-172, the airspeed in Segment 1 turns out to be the same in all 3 segments. In addition, the 65 KCAS just happens to be the best glide speed of a C-172 at gross weight. This result depends on the aerodynamic characteristics of the aircraft being flown. Table 3 shows a comparison of the performance parameters of 16 different aircraft, both fixed gear and retractable. Here, we show the KCAS for both $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$, and both the turn radius and the rate of turn of the aircraft in Segment 1. Thus, Schiff's statement of flying the aircraft at best glide speed or slightly lower is not completely accurate, since Table 3 shows considerable differences between $\mathrm{V}_{1}$ and $\mathrm{V}_{2}$ for some of the aircraft.

Table 3: General Aviation Aircraft Performance Parameters

| Aircraft | V1(Knots) | V2(Knots) | V2-V1 <br> (Knots) | R1(ft) | Turn Rate <br> (deg/sec) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| C-172 | 65 | 65 | 0 | 379 | 16.8 |
| C-177 | 72 | 74 | 2 | 465 | 15.2 |
| C-182 | 73 | 70 | -3 | 478 | 15 |
| PA28-180 | 77 | 76 | -1 | 532 | 14.2 |
| PA28-235 | 85 | 85 | 0 | 648 | 12.8 |
| PA32-300 | 80 | 87 | 7 | 574 | 13.7 |
| SR22 | 97 | 92 | -5 | 844 | 11.3 |
| C-172RG | 75 | 73 | -2 | 505 | 14.6 |
| C-177RG | 80 | 75 | -5 | 574 | 13.7 |
| C-182RG | 74 | 80 | 6 | 491 | 14.8 |
| C-210 | 96 | 88 | -8 | 827 | 11.3 |
| PA28R-201 | 78 | 92 | 14 | 546 | 14 |
| PA24-260 | 88 | 87 | -1 | 695 | 12.4 |
| PA32R-300 | 81 | 80 | -1 | 589 | 13.5 |
| F33 | 85 | 110 | 25 | 648 | 12.8 |
| A36 | 82 | 105 | 23 | 603 | 13.3 |

Thus, depending on the aircraft, one may need to fly a different airspeed in Segments 2 and 3 than is flown in Segment 1. However, if $V_{2}$ is not equal to $V_{1}$, the bank angle in Segment 3 will be limited by the load factor $n=0.83\left(\frac{V_{2}}{V_{S_{18}}}\right)^{2}$, where $V_{S_{18}}$ is the $1-\mathrm{g}$ stall speed of the aircraft. This bank angle ensures that the airspeed flown provides a $10 \%$ pad above the accelerated stall speed at that bank angle. Thus, the maximum bank angle that should be flown in Segment 3 is given by

$$
\begin{equation*}
\operatorname{Cos} \Phi_{\max }=\frac{1}{n}=1.21\left(\frac{V_{S_{1 g}}}{V_{2}}\right)^{2} \tag{19}
\end{equation*}
$$

Using these performance parameters in Table 2, we can derive the altitude loss in each segment as shown in Figure 10 below.


Figure 10: Altitude Loss in 3 Segments of Turnback Maneuver

We observe a region close-in to the DER which is labeled " $\frac{D}{R_{1}}<2$ ", wherein, the 15degree bank would limit the aircraft from rolling out on the runway centerline. We determine the length of this region by requiring the aircraft to initiate a turnback maneuver no closer than the required lead distance $L$. The resulting minimum value of $\left(\frac{\bar{D}}{R_{1}}\right)_{\text {min }}$ is given by

$$
\begin{equation*}
\left(\frac{\bar{D}}{R_{1}}\right)_{\min }=\sqrt{\frac{R_{3}}{R_{1}}} \tag{20}
\end{equation*}
$$

Here we have utilized the shallow flight path approximation (i.e., $\gamma_{g}<12 \mathrm{deg}$ )
Since the airspeed in all three segments is flown at 65 KCAS, the ratio $\frac{R_{3}}{R_{1}}$ is given by

$$
\begin{equation*}
\frac{R_{3}}{R_{1}}=\frac{\operatorname{Tan}\left(\varphi_{1}\right)}{\operatorname{Tan}\left(\varphi_{3}\right)} \tag{21}
\end{equation*}
$$

Thus, for $\varphi_{1}=45$ and $\varphi_{3}=15$ degrees, the minimum value of $\left(\frac{\bar{D}}{R_{1}}\right)$ is 1.93 . Therefore, initiating the turnback maneuver at no closer than twice the radius $\mathrm{R}_{1}$, will accomplish both objectives, i.e. keeping the intercept angle less than 55 degrees to the runway centerline, and having sufficient lead to execute the 15-degree bank in Segment 3, while rolling out over the runway centerline.

The behavior of the altitude loss in each segment is intuitive. In Segment 1, the further from the DER the aircraft is located when initiating the turnback maneuver, the smaller the intercept angle and thus the total number of degrees turned is reduced. In Segment 3, the number of degrees turned is dependent on the intercept angle. Thus, the smaller the intercept angle, the lower the altitude loss in this segment. Finally, Segment 2, which shows the altitude loss in this segment increasing as a function of distance from the DER. Clearly, as the distance from the DER get larger, the altitude loss in Segment 2 becomes the dominant contribution to the total altitude loss. This can be seen in the slope of the curves for both total altitude loss and altitude loss in Segment 2 being nearly identical.

At this point, we have developed the information that answers the question, "How much altitude is needed for a successful turnback maneuver?" Clearly it is a function of distance from the runway. The key question that needs to be answered is: "How does the Pilot use this information in determining whether to attempt a turnback maneuver?" One possibility would have the Pilot setup a waypoint on the GPS corresponding to the DER with the altimeter set to zero at that location. The Pilot or Co-Pilot could monitor the distance from the DER with a callout for a go/no-go for a turnback maneuver during the aircraft climb-out. For example, even if the Pilot had Figure 10 on his kneeboard when the engine failed, the Pilot must make a quick decision under these stressful conditions as to whether the aircraft is located at the distance from the DER that corresponds to the aircraft's height above the DER. A better alternative to mitigate the risk of a performing an "Impossible Turn", would be to have a chart that the Pilot could view prior to departure, which would show when "Never to attempt a turnback maneuver". This chart would show the minimum runway length needed for a PSTM. The Pilot could view the chart on the ground prior to departure and determine whether the runway length was at or greater than the RMRL. If the runway length was at/above the RMRL, the Pilot could monitor the aircraft performance during the takeoff and climb-out, in order to ensure the aircraft performance was consistent with the aircraft performance used to determine the RMRL. The best indicator of the required performance of the aircraft would be the height of the aircraft over the DER. If the height of the aircraft over the DER was at or above its expected value, the Pilot would consider the aircraft performance as "to be expected" and could initiate a turnback maneuver if necessary.

Before delving into how to develop the RMRL chart, we need to discuss another issue that relates to the question: "Should the Pilot attempt a turnback maneuver at a high-density altitude airport?" Consider an airport at a density altitude of 5000 MSL. We can derive a set of similar curves of altitude loss for this case. Here the anticipated parameters that will change are: (1) Altitude loss per degree of turn due to the reduction in the air density, (2) Increasing intercept angle due to the increased radius of the turn in Segments 1, and (3) Increased total number of degrees turned. Figure 11 shows a comparison of the altitude loss in the 3 segments of a turnback maneuver, at both sea level and a 5000-foot density altitude, in the case of a C-172 at gross weight, departing under a no-wind condition.


Figure 11: Effect of Density Altitude on Altitude Loss in the Turnback Maneuver
(Sea Level and 5000 MSL)

We observe the major effect of the increased altitude loss arises from Segments 1 and 3. The largest difference in the total altitude lost occurs close in to the DER, with an increase of about 81 feet. Figure 12 shows the effect of the density altitude on the intercept angle. As can be seen, the maximum change in the intercept angle occurs close in to the DER and is about 6 degrees. In Segment 1, the total turning angle is $180+\Psi$, an increase of 6 degrees in the intercept and has only a small effect in the total
number of degrees turned. The major portion comes from the density effect in $\left(\frac{d h}{d \theta}\right)_{1}$. In Segment 3, the major effect comes from the from both the increase in total number of degrees turned, and the density altitude effect on $\left(\frac{d h}{d \theta}\right)_{3}$.


Figure 12: Effect of Density on Intercept Angle

At this point, we have created all the information necessary to produce the RMRL chart. In Section 4 we move on to Step 3, which outlines the methodology used to generate the RMRL chart.

## 4. Creation of the RMRL Chart

As stated earlier, the key to a successful turnback maneuver is to be sure that the aircraft is at or above the required altitude at the correct distance from the DER. This will depend on the length of the runway and the takeoff/climb profile used for the departure. Figure 13 shows a generic takeoff/climb profile which will be used to develop the RMRL chart. In this profile, the aircraft will start the takeoff roll at the beginning of the runway using the following scenario:
(1) The aircraft performs a short-field takeoff over a 50-foot obstacle at full power and at $V_{x}$.
(2) At 50 feet AGL the aircrafts pitches over and accelerates to a yet to be determined airspeed $V^{*}$, with a corresponding flight path angle $\gamma^{*}$.
(3) The aircraft continues to climb out at $V^{*}$ until at some distance from the DER the engine fails.
(4) The region after the engine failure (approximately 5 seconds) will be used to configure the aircraft to the proper airspeed for Segment 1.
(5) The aircraft initiates the 45-degree bank at a distance $\bar{D}$ from the DER.

## Take-Off/Climb Profile for a Turn-back Maneuver



Figure 13: Generic Takeoff/Climb Profile for the Turnback Maneuver

The RMRL chart requires accumulating the distances $L_{1}$ and $L_{2}$, and the heights from the 50 feet obstacle height to the height at which the aircraft attains the airspeed $V^{*}$. When the engine fails at some distance from the DER, if the aircraft holds the attitude as suggested by Schiff, a Cessna 172 can decelerate as much as 10 knots during the 5 second so-called "Human Factors" region. If the aircraft decelerates below the required airspeed for Segment 1, i.e., $\mathrm{V}_{1}$, the Pilot will need to lower the nose and lose altitude to accelerate to $\mathrm{V}_{1}$. Note that at the point of engine failure, the loss of thrust will not allow the aircraft to gain the additional altitude expected at the distance $\bar{D}$ from the DER. This lost altitude is given by $L_{H} \operatorname{Tan} \gamma^{*}$. In addition, due to non-optimal Pilot skills, the aircraft may fall below the altitude of the aircraft at the time of the engine failure, i.e., an additional altitude loss of $\Delta h_{\text {pilot }}$. However, in this analysis we will assume $\Delta h_{\text {pilot }}=0$, and thus, the aircraft will be at the altitude corresponding to the point at which the engine failure occurred. One does not consider this a safety factor, but as an estimated correction to the aerodynamic model.

After engine failure, the forces decelerating the aircraft are the drag and the component of gravity acting backward along the original flight path corresponding to $\gamma^{*}$. Figure 14 shows the forces in a steady climb, while Figure 15 shows balance of forces along and perpendicular to the flight path.


Figure 14: Forces in a Steady Climb

## Forces in Steady Climbing Flight

- Balance of forces along the flight path

$$
\begin{aligned}
& \mathbf{T}=\mathbf{D}+\mathbf{W} \operatorname{Sin} \gamma \\
& \operatorname{Sin} \gamma^{\circ}=\frac{(\mathbf{T}-\mathbf{D})}{\mathbf{W}}
\end{aligned}
$$

- Balance of forces perpendicular to the flight path

$$
\mathbf{L}=\mathbf{W} \operatorname{Cos} \gamma \quad \begin{gathered}
\text { Lift is always less than weight } \\
\text { in a steady climb }
\end{gathered}
$$

Figure 15: Balance of Forces in a Steady Climb

The flight path angle $\gamma^{*}$ depends on the difference between the thrust at the propeller and the drag. When the thrust is lost due to the engine failure, the net forces along the flight path, which is composed of the drag and the component of the weight of the aircraft acting backwards along the flight path causes the aircraft to decelerate. How fast the aircraft decelerates over the 5 seconds depends on the magnitude of the drag and the component of the weight along the flight path. As mentioned above, a C-172 can decelerate 10 knots over the 5 second time-period. The selection of the climb speed $V^{*}$ will depend on how much the airspeed is reduced. If at the time of engine failure, the aircraft is immediately pitched down to level flight attitude, the deceleration due to the gravity component will be reduced and the aircraft will not slow down as much. However, with proper training in the stick and rudder skills, we will assume the aircraft slows down by 5 knots, and thus, the selection of the airspeed for the climb phase will be determined by $V^{*}=V_{1}+5$.

Since we have determined $V^{*}$, the requirement is to determine the corresponding climb angle $\gamma^{*}$. However, what is really needed is a chart showing the climb angle versus calibrated airspeed, as a function of altitude. The propeller data is used to develop this type of chart. We have taken Figure 16 from the Ref. 1, "Performance of Light Aircraft" by Lowry, wherein a "Bootstrapping Method" was employed to back out this data over several C-172 flights. The "Bootstrapping Method", utilizes the correct form of the aerodynamic parameters which contain unknown constants corresponding to the aircraft flown. One then flies the aircraft under different flight scenarios to record
the aircraft performance, and thus back-out the unknown constants. Although Figure 16 corresponds to a 160 HP C-172 with a gross weight of 2400 lbs ., the performance data for the aerodynamic model was taken from a 160 HP C-172 with a gross weight of 2300 lbs. However, we will use this chart to help estimate the climb angle for the 1977 C-172 at $V^{*}$. Note that both aircraft utilize the identical propeller.

## C-172 Climb Angle Versus Airspeed (Gross Weight)


"Performance of Light Aircraft " by J.T. Lowry
Figure 16: Climb Angle versus Calibrated Airspeed

To determine the climb angle for $V^{*}=70$ KCAS, we use the POH to determine the maximum rate of climb at the corresponding density altitude of the airport. For example, in the case of a sea level departure at standard temperature, the maximum rate of climb is 770 feet per minute, at an airspeed of 73 KCAS. Since the rate of climb is given by $R C=V_{1} \operatorname{Sin} \gamma^{*}$, we determine the climb angle to be 5.98 degrees. Using Figure 16 to determine the variation in the sea level climb angle versus calibrated airspeed, we see that at 70 KCAS, $\gamma^{*}$ would be approximately 6.5 degrees. This would be the climb angle used for determining the RMRL for the $\mathrm{C}-172$ aircraft.

However, without Figure 16, one would need to develop the identical chart. It is important for all Pilots to have a basic understanding of how Figure 16 is created, since this is the key to producing the RMRL chart. The procedure is described below.

Propeller aerodynamic theory provides a simple relationship for the thrust produced by the propeller. This is given by

$$
\begin{equation*}
T=C_{T} \rho n^{2} d^{4} \tag{22}
\end{equation*}
$$

Where $\mathrm{C}_{\mathrm{T}}$ is the thrust coefficient, $\rho$ the air density, n is the propeller revolutions per sec , and d is the diameter of the propeller. Thus, for any propeller, one needs the thrust coefficient to determine the propeller thrust at any value of $n$ and air density. We can rewrite the formula for the climb angle as

$$
\begin{equation*}
\operatorname{Sin} \gamma_{C}=\frac{\left[C_{T} \rho n^{2} d^{4}-D\right]}{W} \tag{23}
\end{equation*}
$$

Where we have replaced $\gamma^{*}$ with $\gamma_{c}$ so that the reader better understands it represents the climb angle during the climb-out.

One can express the drag as

$$
\begin{equation*}
D=C_{D} S \frac{1}{2} \rho V^{2} \tag{24}
\end{equation*}
$$

Thus, the climb angle can be expressed as

$$
\begin{equation*}
\operatorname{Sin} \gamma_{C}=\frac{\left[2\left(\frac{C_{T}}{J^{2}}\right)\left(\frac{d^{2}}{S}\right)-C_{D}\right] S \rho V^{2}}{2 W} \tag{25}
\end{equation*}
$$

Equation (25) contains a new parameter, J , which is the propeller advance ratio, and is shown in magenta. It is the ratio of the forward velocity of the aircraft to the propeller tip speed and is given by

$$
\begin{equation*}
J=\frac{V}{n d} \tag{26}
\end{equation*}
$$

Using eq. (11) for the drag coefficient, we can now express the climb angle as

$$
\begin{equation*}
\operatorname{Sin} \gamma_{C}=\frac{\left[2\left(\frac{C_{T}}{J^{2}}\right)\left(\frac{d^{2}}{S}\right)-\left(C_{D_{0}}+k C_{L}^{2}\right)\right] S \rho V^{2}}{2 W} \tag{27}
\end{equation*}
$$

The value of $C_{D_{0}}$ used in eq. (27), should correspond to the engine idling value, rather than the propeller windmilling value, as shown in Table 1. The lift coefficient, $\mathrm{C}_{\perp}$ can be determined by $L=W \operatorname{Cos} \gamma_{c}$. However, in the case of shallow climb angles, one can set $L \simeq W$, and thus the lift coefficient is determined by

$$
\begin{equation*}
C_{L}=\frac{2\left(\frac{W}{S}\right)}{\rho_{s l} V_{C}^{2}} \tag{28}
\end{equation*}
$$

Here we express the lift coefficient in terms of calibrated airspeed $\mathrm{V}_{\mathrm{c}}$, rather than TAS. It is best to express the climb angle in terms of the calibrated airspeed, i.e.,

$$
\begin{equation*}
\operatorname{Sin} \gamma_{C}=\frac{\left[2\left(\frac{C_{T}}{J^{2}}\right)\left(\frac{d^{2}}{S}\right)-\left(C_{D_{0}}+k C_{L}^{2}\right)\right] \rho_{s l} V_{C}^{2}}{2\left(\frac{W}{S}\right)} \tag{29}
\end{equation*}
$$

For consistency, we can also express J in terms of calibrated airspeed, i.e.,

$$
\begin{equation*}
J=\frac{\sqrt{\frac{\rho_{s l}}{\rho}} V_{C}}{n d} \tag{30}
\end{equation*}
$$

Thus, when the climb angle is expressed by eq. (29), the density effect only enters through the J parameter, which in turn affects the thrust coefficient.

To determine the climb angle, we will need to determine $\mathrm{C}_{\mathrm{t}}$ and J . Although we know V and d , in the case of a fixed-pitch propeller, the value of n is not known before hand, since it is determined by the equilibrium of the torque being delivered by the crankshaft and that being absorbed at the propeller by the aerodynamic forces in the plane of rotation of the propeller. However, if one flies the aircraft at a particular calibrated airspeed, the Pilot can read the tachometer and determine the propeller rotation rate in rev/sec. With this knowledge, one can compute the propeller advance ratio. Thus, what is left to determine is the thrust coefficient $\mathrm{C}_{\mathrm{t}}$.

Using propeller aerodynamics, we can determine three important propeller parameters. These are: (1) Thrust coefficient $C_{T}$ (2) Power coefficient $C_{p}$ and (3) Propeller efficiency $\eta$, all as a function of the propeller advance ratio J. Figure 17, taken from Lowry, shows the propeller characteristics for a McCauley 7557 propeller, installed on a 1977 C-172.


Figure 17: Propeller Characteristics for McCauley 7557 on C-172
Using eq. (27), we can determine the climb angle at any particular calibrated airspeed, density altitude and aircraft weight. Again, we observe the key parameters in the magnitude of the climb angle are the aircraft weight and air density. Both higher aircraft weight and density altitude reduce the climb angle. The reader now has the knowledge to develop Figure 16 for the aircraft being flown. At this point, we have all the information necessary to develop the RMRL.

For the reader unable to obtain the propeller characteristics for their aircraft, one approach is to flight test the aircraft to obtain the data. The method utilized will require a GPS onboard the aircraft. We describe the method below:
(1) Set a waypoint for the runway centerline at the DER
(2) Set the altimeter to read zero at this waypoint
(3) Using the POH , determine $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{Y}$ (KCAS) for the density altitude of the departure airport
(4) Determine at least 4 airspeeds for $V^{*}$, equally spaced between best angle and best rate
(5) Define a takeoff/climb profile you intend to use on all your takeoff's
(6) Set the runway course in the GPS so the Pilot can track the centerline of the runway
(7) Once the aircraft is established at $V^{*}$, trim the aircraft and maintain $V^{*}$ up to 1200 AGL while tracking the centerline of the runway. Also note the aircraft groundspeed and in the case of a fixed-pitch propeller, the RPM on the tachometer
(8) Note the altitude when the aircraft is over the DER (i.e., over the DER waypoint)
(9) Record the aircraft altitude and groundspeed every 0.1 nm on the GPS all the way up to 1200 AGL
(10) Plot the altitude versus distance from the DER in feet (i.e. 0.1 nm is approximately 600 feet)
(11) Draw a best fit straight line through the data points
(12) The approximate climb angle is given by the inverse tangent of the slope of the line

This procedure should provide a reasonable estimate of the climb angle during the climb phase of the departure. However, if the winds are not calm during the above flight experiment, a correction for the wind can be obtained using the methodology discussed in Appendix A.

The method that is used to determine the RMRL is based on first determining the required height of the aircraft over the DER, $h_{D E R}$. The second step is to generate a curve which shows the altitude of the aircraft above the DER as a function of the runway length. Using the determined value of $h_{\text {DER }}$, the RMRL can be read off the chart. Thus, the key step in producing the RMRL chart is to develop a method to determine $h_{\text {DER }}$.

We start by expressing the height of the aircraft above the DER as a function of the distance from the DER, i.e., $\bar{D}$. Thus,

$$
\begin{equation*}
h=h_{D E R}+\bar{D} \operatorname{Tan} \gamma_{C} \tag{31}
\end{equation*}
$$

Here, $\gamma_{c}$ is the climb angle corresponding to the calibrated airspeed $V^{*}$. However, recall from Figure 13 , there is a region $L_{H}$, which corresponds to a 5 -second delay before initiating the turnback maneuver. During this time, the aircraft does not have the thrust to continue the climb gradient, and while configuring the aircraft to $\mathrm{V}_{1}$, the aircraft may drop below the altitude at the point of the engine failure. However, we will assume that $\Delta h_{\text {pilot }}=0$, so that the aircraft descends to the altitude at which the engine failure initially occurred. Thus, the aircraft will need to recapture this lost altitude by the time it reaches the DER. Therefore, we can express this by modifying eq. (31) as follows:

$$
\begin{equation*}
h=h_{D E R}+\bar{D} \operatorname{Tan} \gamma_{c}-L_{H} \operatorname{Tan} \gamma_{c} \tag{32}
\end{equation*}
$$

In the case of shallow climb angles (i.e. less than $\approx 12$ degrees), we can express $L_{H}$ as

$$
\begin{equation*}
L_{H}=\frac{1}{2} \sqrt{\frac{\rho_{S L}}{\rho}}\left(V^{*}+V_{1}\right) t_{H} \tag{33}
\end{equation*}
$$

Where $V^{*}$ is the calibrated airspeed for the climb, $\mathrm{V}_{1}$ is the calibrated airspeed for Segment 1 , and $t \boldsymbol{t}$ is the 5 -second time between engine failure and time for initiating the turnback maneuver. However, with proper stick and rudder skills, this time $t_{H}$, can be reduced to approximately 3 seconds. Note that the density ratio shown in magenta in eq. (33), converts the calibrated airspeed to TAS.

We then require, that at each distance $\bar{D}$ from the DER, for which the aircraft initiates a turnback maneuver, the expected altitude loss, $h_{a}$, must be equal to the altitude given in eq.(32). The expected altitude loss in the turnback maneuver as determined by the aerodynamic model is given by

$$
\begin{equation*}
h_{a}=(180+2 \Psi)\left(\frac{d h}{d \theta}\right)_{1}+\left[\left(\frac{d h}{d \theta}\right)_{3}-\left(\frac{d h}{d \theta}\right)_{1}\right] \Psi+\left(\bar{D}-L_{0}\right) \operatorname{Tan} \gamma_{g} \tag{34}
\end{equation*}
$$

Here $L_{0}$ is the distance from the DER where the Segment-3 turn is initiated, and is given by eq.(17). Equating eq. (32) and (34), we obtain the following equation for hDER,

$$
\begin{equation*}
\left.\left.\left.h_{D E R}=(180+2 \Psi) \frac{d h}{d \theta}\right)_{1}+\Psi\left[\frac{d h}{d \theta}\right)_{3}-\frac{d h}{d \theta}\right)_{1}\right]+\bar{D}\left(\operatorname{Tan} \gamma_{g}-\operatorname{Tan} \gamma_{c}\right)+\left(L_{H} \operatorname{Tan} \gamma_{c}-L_{0} \operatorname{Tan} \gamma_{g}\right) \tag{35}
\end{equation*}
$$

Equation (35) conveys some interesting information. We see that as $\bar{D}$ gets large, depending on the sign of the quantity ( $\operatorname{Tan} \gamma_{g}-\operatorname{Tan} \gamma_{c}$ ), hder will increase if the sign is positive, and decrease, if the sign is negative. Thus, if the glide angle is greater than the climb angle, as we get further away from the DER, the aircraft will require more altitude over the DER for a PSTM. Since the glide path angle is independent of altitude, one would expect such a variation when departing a high-density altitude airport, where the climb angle is reduced due to the reduction in the thrust. When departing from sea level airports, one would expect the required value of hder to decrease as $\bar{D}$ gets large. This effect will be observed when discussing the effect of density altitude on the turnback maneuver. In addition, when $\gamma_{c}=\gamma_{g}$, as $\bar{D}$ gets large, the intercept angle approaches zero, and thus the height above the DER approaches a constant, which is given by

$$
\begin{equation*}
\left.h_{D E R}=180 \frac{d h}{d \theta}\right)_{1}+L_{H} \operatorname{Tan} \gamma_{c} \tag{36}
\end{equation*}
$$

To validate the Schiff ROT, item (1), we need to obtain the fraction of the OAL, i.e.,

$$
\begin{equation*}
\mathrm{OAL}=\left(\frac{d h}{d \theta}\right)_{1}(360) \tag{37}
\end{equation*}
$$

Equation (37) arises when one investigates the limiting value of hDER as $\frac{\bar{D}}{R_{1}}$ approaches unity. This represents the "270-90" turnback scenario shown in Figure 3. In this case, $\left.\left.R_{1}=R_{3}, \frac{d h}{d \theta}\right)_{1}=\frac{d h}{d \theta}\right)_{3}$, and $\bar{D}=L_{0}$. Thus, in this limit, hDER becomes

$$
\begin{equation*}
\left.h_{D E R}=(180+2 \Psi) \frac{d h}{d \theta}\right)_{1}+\left(L_{H}-R_{1}\right) \operatorname{Tan} \gamma_{c} \tag{38}
\end{equation*}
$$

Since, in this scenario, the intercept angle $\Psi$ approaches 90 -degrees, we see that

$$
\begin{equation*}
\left.h_{D E R}->360 \frac{d h}{d \theta}\right)_{1}+\left(L_{H}-R_{1}\right) \operatorname{Tan} \gamma_{c} \tag{39}
\end{equation*}
$$

Note, eq.(39) contains an additional term, containing both $\mathrm{L}_{\text {н }}$ and $\mathrm{R}_{1}$. This term is usually small compared to the first term. As an example, in the case of a C-172, this term is approximately 19 feet, and thus, one can use the first term in eq. (39) as a reference for specifying the value of hDER as a fraction of the OAL.

Figure 18 shows the required height of the aircraft over the DER as a function of distance from the DER. We also show hDER as a fraction of the "Observed Altitude Loss", so the reader can relate these results back to the Schiff ROT. It is important to understand that it is the value of hDER that is the key to determining whether it is a PSTM, since the decision to turn back will be based on the RMRL chart. If the performance of the aircraft is not as anticipated, and/or the Pilot does not properly fly the aircraft during the climb phase, the aircraft will arrive over the DER at a lower altitude, and thus, the Pilot should not initiate the turnback maneuver.


Figure 18: C-172 Required Height over DER for a PSTM
(Sea Level, Gross Weight, No Wind)

Now that we have created Figure 18, it will be necessary to create a chart which shows the height of the aircraft over the DER as a function of runway length. This chart requires the takeoff/climb profile shown in Figure 13. The corresponding chart for a C172 at sea level, gross weight, and with no wind, is shown in Figure 19. To generate the RMRL, the user selects the distance from the DER and then obtains the value of hder from Figure 18. Using this value of hder, the user enters Figure 19 and determines the RMRL. Figure 20 shows the resultant RMRL chart obtained using the above procedure. Although, with proper training one could conceivably execute a PSTM at $\frac{\bar{D}}{R_{1}}=1$, we assume that no turnback maneuvers are to be initiated before $\frac{\bar{D}}{R_{1}}=2$, and thus the
charts show results for $\frac{\bar{D}}{R_{1}}>2$. Again, $\frac{\bar{D}}{R_{1}}=2$, under a no wind condition would allow for a maximum intercept angle of no more than 55 degrees, and will also allow for the proper lead distance for the Pilot to roll out of the turn over the runway centerline when using a bank angle of 15 degrees in Segment 3.


Figure 19: Expected Height of Aircraft versus Distance from Beginning of Runway (Sea Level, Gross Weight, No Wind)

Required Minimum Runway Length ( $D / R_{1}>2$ )


Figure 20: RMRL for a Potentially Successful Turnback Maneuver (C-172 at Gross Weight, Sea Level, and No Wind Condition)

Figure 20 shows some interesting trends. First, the RMRL is not constant. As the value of $\bar{D}$ increases, the RMRL decreases. To understand this trend, we need to go back to Figure 11, which shows the altitude loss in each of the 3 segments. Note that as we start from $\bar{D}=800$ feet and move outward, the altitude loss in Segments 1 and 3 are decreasing due to the intercept angle decreasing. However, after a certain distance from the DER, the altitude loss in Segment 2 becomes the dominant contributor to the total altitude loss. The RMRL decreases since the magnitude of the climb angle is slightly larger than the magnitude of the glidepath angle. In this case, the aircraft is gaining more altitude climbing outbound than losing coming back, and thus, the aircraft does not require as long a runway if the turnback maneuver is initiated further out.

To understand how to utilize Figure 20, we have added three dashed lines corresponding to three different runway lengths at the departure airport. The red line corresponding to a runway length of 3100 feet is completely below the RMRL curve, and thus, the runway is too short to perform the turnback from any distance from the DER. Obviously, here the decision would be not to attempt a turnback maneuver. In this scenario, the best option would be to land straight ahead, plus/minus 30 degrees (i.e.,
assuming no terrain issues). However, in the case of a 3750-foot runway, the Cyan line cuts through the curve at 2200 feet from the DER, indicating that beyond this distance from the DER, a PSTM is possible. However, this situation alerts the Pilot about attempting a turnback too close to the DER. Finally, if the runway length is 4500 feet, the green line is above the RMRL curve, and thus, a PSTM may be realized beyond 800 feet from the DER. Thus, the risk is mitigated by making the decision on the ground prior to departure. In addition, one should realize that any time the RMRL is less than the departing runway length, there will be an excess of altitude when the aircraft completes Segment 3 over the runway. The excess altitude $h_{\text {excess }}$ is given by

$$
\begin{equation*}
h_{\text {excess }}=(L-R M R L) \tan \gamma_{c} \tag{40}
\end{equation*}
$$

Where $L$ is length of the departure runway. Thus, if the Pilot departs from a 4500-foot runway and initiates a turnback maneuver at 3300 feet from the DER, the aircraft will be 107 feet above DER when the aircraft rolls wings level over the runway centerline. It is important that the Pilots are aware of this prior to departure so they can anticipate the possibility of having to dissipate this altitude by adding flaps or slipping the aircraft when approaching the DER.

Another important area of concern relates to Pilots executing a turnback maneuver at a high-density altitude airport. Very little information is ever provided by the pundits on this subject. Figure 21 shows the takeoff/climb characteristics of the C-172 departing from an airport with a 5000-foot density altitude. In this scenario, the airspeed flown was 70 KCAS and the climb angle was determined to be 4.32 degrees. It is easy to see the consequences of the lower climb angle at the 5000-foot density altitude as compared to the sea level departure. Figure 22 shows the effect of density altitude on the RMRL. Note, as the distance from the DER increases, there is a continual decrease in the RMRL for departures at sea level, however, when departing from a 5000-foot density altitude airport, the RMRL turns around and increases beyond 2200 feet from the DER. Recall that the difference between the climb angle and the glide angle at sea level is +0.2 degree, however, at 5000 -foot density altitude, the difference is -2 degrees. This reversal in sign of the difference, manifests itself as an increase in the RMRL as we initiate the turnback further from DER. In addition, since we are not considering turning back closer than twice the radius of the turn in Segment 1, the distance from the DER to initiate the turnback gets pushed out from the DER due to the radius of the turn in Segment 1 increasing, while at the same time, the farthest distance from the DER that one would turnback is limited by the runway length at the airport. Thus, the range of distance for a PSTM is being squeezed into a narrow region. This can be somewhat risky, and that is why the author does not recommend executing a turnback maneuver at a high-density altitude airport in an aircraft with a relatively high power-loading, such as a C-172 at gross weight.


Figure 21: Effect of Density Altitude on Height above the DER (Gross Weight, No Wind)

Effect of Density Altitude on RMRL


Figure 22: Effect of Density Altitude on RMRL (Gross Weight, No Wind)

Figure 23 shows the effect of density altitude on the required height of the aircraft above the DER. In addition, we have also plotted the fraction of the OAL. Note that at the higher density altitude departure, the required height above the DER first decreases and then increases. This is due to the sign of the difference between the climb angle and glide angle changing sign. This also translates to the higher fraction of the OAL, which as one would expect, follows the required height over the DER.

In Section 5 we compare the results of the aerodynamic turnback model with the Schiff ROT, for when to initiate the turnback maneuver.


Figure 23: Effect of Density Altitude on Height over DER and Fraction of OAL (Gross Weight and No Wind)

## 5. Comparison of the Aerodynamic Model with Schiff's Rule-ofThumb for the Teardrop Turnback Maneuver

Now that we have developed Figures 21-23 for a C-172, we can start to validate Schiff's ROT for when to attempt a turnback maneuver. First, let's recall the Schiff ROT. In the Schiff experiment, he states "Do not consider turning back unless both (1) and (2) below are satisfied."
(1) The aircraft has reached at least $2 / 3$ of the "Observed Altitude Loss (OAL)" when passing over the DER, and
(2) It has reached at least the "Minimum Turnback Height".

Since both (1) and (2) must be satisfied together, we will first consider (1). Since "at least" means it should be a PSTM when the height over the DER is $2 / 3$ of the OAL, and thus, we assume the height over the Der is $2 / 3$ of the OAL. In the case of a C-172, the OAL is given by

$$
\begin{equation*}
O A L=360\left(\frac{d h}{d \theta}\right)_{1} \tag{41}
\end{equation*}
$$

Where the quantity $\left(\frac{d h}{d \theta}\right)_{1}=1.08$ for a C-172 at sea level and gross weight. Thus, the value of hobs=389 feet. Satisfying (1) in Schiff's Rule-of-Thumb, requires the height over the DER to be $2 / 3^{*} h_{\text {obs, }}$ which is equal to 259 feet. Figure 24a shows the Schiff's ROT as applied to a C-172 departing an airport at sea level density altitude, and with no wind. We have added a series of lines which represents various climb angles of the aircraft used in the takeoff/climb profile. The intersection of the lines of constant climb angle with the Schiff "Turnback Height" shows at what distance from the DER the aircraft can initiate the turnback maneuver. In the case of a C-172, the climb angle is 6.5 degrees (i.e., the magenta line). Thus, the application of the Schiff ROT would indicate that the turnback maneuver should not be initiated prior to the point designated as TBI (i.e., turnback initiation), which is 2800 feet from the DER. There is no other information provided by Schiff in terms of any limitations on when the ROT should be utilized.

## Interpretation of Schiff Rule-of-Thumb



Figure 24a: Schiff's Rule-of-Thumb for C-172 (Sea Level, Gross Weight, No Wind)

Figure 24b shows the identical information as 24 a, except we have deleted all the lines for other possible climb angles. We have also added one additional curve that represents the EAL in the turnback maneuver as a function of distance from the DER (i.e., the black line), as predicted by the aerodynamic model of the turnback maneuver. Notice that the 6.5-deg climb angle line crosses the EAL curve at 1400 feet from the DER, and thus, the aircraft would have sufficient altitude to initiate a turnback maneuver at any distance beyond 1400 feet. However, the Schiff ROT informs the Pilot not to initiate the turnback maneuver prior to 2800 feet from the DER. Therefore, the Schiff ROT dismisses all PSTM's between 1400 and 2800 feet from the DER.


Figure 24b: Schiff's Rule-of-Thumb for C-172 (Sea Level, Gross Weight, No Wind)

One might ask the question, "What height over the DER is necessary to realize a PSTM for all distances greater than twice $R_{1}$ ? The obvious answer to the question is to increase the height of the aircraft over the DER. The result of this modification is shown in Figure 24c. Here, we have raised the value of hDER to 314 feet to clear the EAL curve at a distance equal to twice $R_{1}$. In this case, a PSTM is realized beyond twice $R_{1}$, provided hDER is $82 \%$ of the OAL. Thus, at sea level, the Schiff ROT is overly restrictive in that it dismisses all PSTM's at distances between approximately 800 and 2800 feet from the DER.


Figure 24c: Schiff's Rule-of-Thumb for C-172 (Sea Level, Gross Weight, No Wind)

We now repeat the same process for the scenario of a C-172 departing an airport with a density altitude of 5000 feet, at gross weight, no wind, and see how Schiff's ROT fairs in this case. Figure 25 shows the equivalent chart as in Figure 24c. Note, using the Schiff ROT with the 4.32 -deg climb angle predicts a PSTM at 5100 feet from the DER. However, one observes that it is completely below the EAL curve out to 6500 feet from the DER. It is necessary to increase the aircraft height above the DER from 301 to 430 feet, to allow for a PSTM between 800 and 4700 feet from the DER. The value of the OAL at a density altitude of 5000 feet is 451 feet. Thus, the fraction of the OAL corresponding to the above range of distances from the DER is 0.95 .


Figure 25: Schiff Rule-of-Thumb for C-172 (5000 MSL Altitude, Gross Weight, No Wind)

Thus, Schiff's ROT clearly does not provide the correct result for both sea level and 5000 -foot density altitude turnback maneuvers. In these scenarios, it is better to drop (2) in the ROT and increase the fraction of the OAL to $100 \%$. We should also point out that the aerodynamic model provides an additional increment of altitude over the DER due to the loss of thrust 5 seconds before initiating the turnback maneuver. Recall that the height that is added back in over the DER is $L_{H} \operatorname{Tan} \gamma_{c}$.

The above results are dependent on the specific aircraft climb angle and glide angle. Rather than utilizing a ROT for determining the "Turnback Height", it is much simpler to just use the aerodynamic model of the turnback maneuver. The major advantages of such a tool are:
(1) Aircraft specific
(2) Properly corrects for effects of density altitude and aircraft weight
(3) Takeoff/Climb profiles can be traded in order to determine the best climb speed $V^{*}$, and climb angle $\gamma_{c}$
(4) Provides the RMRL chart for determining a PSTM from the departure airport during the preflight procedure.
(5) Although safety factors have not been applied to the aerodynamic model in this paper, they can be added to account for both uncertainty in the aerodynamic coefficients and non-optimal Pilot skills.

It's clear that the Schiff ROT lacks the physical reality of the geometry of the turnback maneuver, since it neglects any information about Segment 2, i.e., the wingslevel portion of the glide. Summarizing the deficiencies of the model:
(1) The Schiff ROT does not consider the effects of aircraft weight and density altitude in the scaling of the OAL determined by the Pilot under a single set of conditions.
(2) The $150 \%$ factor used by Schiff dismisses PSTM's closer to the DER.
(3) The ROT does not consider the altitude loss in the wings level glide in the determining the conditions under which a PSTM can be realized.
(4) Schiff does not discuss the effect of density altitude on the turnback maneuver, and how it impacts his ROT. However, applying Schiff ROT to high density altitude turnback maneuvers, show that it does not correctly predict the PSTM.
(5) It is assumed that satisfying the two requirements in Schiff's ROT would guarantee a PSTM beyond the point at which no turnback would be initiated. However, if the magnitude of the climb angle is smaller than the glide angle, the RMRL would need to increase. Thus, there may be a distance from the DER beyond which the runway length is too short to execute a PSTM, even with the aircraft attaining $100 \%$ of the OAL over the DER. This situation clearly occurs when operating aircraft with high power loadings at gross weight and at high density altitude airports.

Since the aerodynamic model assumes the turnback maneuver will not be initiated prior to a distance $2 R_{1}$ from the DER, one can determine a conservative estimate of the time beyond the DER at which this distance occurs. In the case of shallow climb angles (i.e., less than 12 deg), a conservative approximation for the time is given by

$$
\begin{equation*}
t_{t b i}=\frac{2 \sqrt{\frac{\rho_{s l}}{\rho}} V_{1}^{2}}{g V^{*}} \tag{42}
\end{equation*}
$$

Where $\mathrm{g}=32.174 \mathrm{ft}^{\wedge} 2 / \mathrm{sec}, \frac{\rho_{s l}}{\rho}$ is the density ratio, and both $V^{*}$ and $\mathrm{V}_{1}$ are the calibrated airspeeds in ft/sec. We can simplify the above formula as follows:

$$
\begin{equation*}
t_{t b i}=\frac{\sqrt{\frac{\rho_{s l}}{\rho}} V_{1}^{2}}{9.53 V^{*}} \tag{43}
\end{equation*}
$$

Here, $V_{1}$ and $V^{*}$ are KCAS.
For example, in the case of a C-172 climbing out at 70 KCAS at a sea level density altitude, the time between passing over the DER and the initiation of the turnback maneuver would be approximately 7 seconds. Thus, if the Pilot started timing over the DER, after 7 seconds, one could initiate the turnback maneuver provided the correct height above the DER had been confirmed.

## 6. Conclusions

We have developed a steady-steady aerodynamic model of the teardrop turnback maneuver under no wind conditions. The no wind case provides a conservative estimate of the altitude loss during the turnback maneuver as a function of the distance from the departure end of the runway (DER). Using the geometric properties of the teardrop turnback maneuver, we show the maneuver is composed of 3 segments. In Segment 1, the aircraft is in a gliding turn to a heading that points the nose of the aircraft directly at the DER. In Segment 2, the aircraft is in a wings-level glide heading directly at the DER, and in Segment 3, the aircraft is in a final gliding turn to align the aircraft directly over the centerline of the runway. The analysis also shows that under a no wind condition, the intercept angle to the runway in Segment 2, is only a function of the ratio $\frac{\bar{D}}{R_{1}}$, where $\bar{D}$ is the distance from the DER at which the turnback is initiated, and $R_{1}$ is the radius of the turn in Segment 1. To avoid large intercept angles, i.e. greater than 55 degrees, we need to avoid initiating a turnback prior to $\frac{\bar{D}}{R_{1}}=2$. Due to the low altitude that the aircraft is being flown at in Segment 3, we assume the bank angle is programmed to be initially 15 degrees. However, there is sufficient margin above the accelerated stall speed to increase the bank angle in Segment 3 to 45 degrees, to avoid both overshooting the runway centerline, and entering an accelerated stall.

In the gliding turn segments, the goal is to minimize the altitude loss per degree of turn, while in the wings-level glide, we attempt to minimize the altitude loss per distance travelled. Using the information in the POH , we describe the procedure used to determine the altitude loss in the turnback maneuver as a function of distance from the DER where the turnback is initiated. A C-172 was selected to demonstrate how to determine this information. These results answer the question of "How much altitude is needed for a "Potentially Successful Turnback Maneuver (PSTM)?" The analysis shows
that during the gliding turn segments, the altitude loss per degree of turn is a function of four parameters. These are: (1) Wing loading, (2) Bank-angle, (3) air density, and (4) $C_{L}(L / D)$. We show that the minimum altitude loss per degree of turn occurs when the bank angle is between 45 and 46 degrees, with the aircraft operating just above the stall angle-of-attack. In the wings-level glide, the aircraft is flown at the maximum L/D ratio to minimize the altitude loss per distance travelled. In the case of a C-172, we show that the appropriate speed to fly all 3 segments is 65 KCAS, which just by coincidence, happens to be the best glide speed in a C-172 at gross weight. This speed also provides a 10\% safety factor above the accelerated stall speed when flying Segments 1 and 3.

To mitigate the risk in attempting the "Impossible Turn", we use the calculated altitude loss as a function of distance from the DER, to develop a "Required Minimum Runway Length (RMRL) chart. This chart shows how much runway is necessary for a PSTM. This chart is extremely useful since the Pilot can compare the departure runway length with the RMRL and determine whether a PSTM is possible. Thus, the decision "When Never to Attempt a Turnback Maneuver", is decided on the ground prior to departure.

A comparison of the results of the aerodynamic model of the turnback maneuver with Schiff's ROT has been made. We show that the Schiff method does not characterize the geometry of the turnback maneuver properly, since is does not include any information on the altitude loss in Segment 2. Using the C-172 for the comparison, we show that the Schiff ROT is overly restrictive, in that it dismisses PSTM's closer in to the DER when the turnback maneuver is executed at a sea level airport. The aerodynamic model shows that raising the height of the aircraft over the DER to 82\% of the "Observed Altitude Loss" allows for a PSTM beyond a distance from the DER equal to $2 \mathrm{R}_{1}$.

When the turnback maneuver is executed at a 5000-foot density altitude airport, the Schiff Rule-of-Thumb indicates a "Possible Turn" beyond 5100 feet from the DER. However, the aerodynamic model shows the "Impossible Turn" from the DER out to 6500 feet. We also show that by increasing the height over the DER to $95 \%$ of the "Observed Altitude Loss", allows for a PSTM from between $2 R_{1}$ and 4700 feet from the DER. Beyond 4700 feet from the DER, the height of the aircraft over the DER needs to increase to $115 \%$ of the "Observed Altitude Loss". This is due to the glide path angle being greater than the climb angle at this density altitude. In this scenario, we show that extremely long runways are required, and the region for which a PSTM can occur is narrow. Thus, one should avoid turnback maneuvers at high density altitude airports when flying aircraft at gross weight with a relatively high-power loading.

In addition, the aerodynamic model uses a somewhat different takeoff/climb profile than the Schiff experiment, with a $V^{*}$ closer to $\mathrm{V}_{Y}$ rather than midway between $\mathrm{V}_{\mathrm{X}}$ and $\mathrm{V}_{\mathrm{Y}}$. This is due to $\mathrm{V}_{1}$ being almost identical to $V^{*}$, and thus, after an engine failure, requiring the aircraft to give up some altitude to configure the aircraft at $\mathrm{V}_{1}$ airspeed prior
to initiating the turnback maneuver. Thus, using a value of $V^{*}=65$ KCAS may have a slight improvement in the climb angle, but the additional altitude lost in configuring the aircraft back to $\mathrm{V}_{1}$ may offset this advantage. However, the aerodynamic model can perform these trade studies.

Although the paper discusses the performance of the C-172 at gross weight, the effect of a reduction in the weight of the aircraft is to increase the climb angle (see eq. (29)) and decrease the OAL (see eq.(5) Thus, one would expect a lower RMRL for a PSTM when flying the aircraft at a reduced weight. Note, to fly the same angle-of-attack at the reduced weight, the aircraft will need to be flown at a reduced airspeed in each of the three segments of the turnback maneuver. This translates to a reduction in turn radius and thus, lower intercept angles.

The author recommends a formal standardized training program for those interested in becoming skilled in flying the turnback maneuver. This program should include at a minimum:
(1) Aerodynamics of the turnback maneuver, including creating both the expected altitude loss and the RMRL charts for the aircraft being flown
(2) Stick and rudder skills in performing the turnback maneuver
(3) Aeronautical decision making and risk mitigation prior to performing the turnback maneuver.

Finally, the most important information that should be taken away from this White Paper in regard to the turnback maneuver, is that although it is a relatively simple geometric maneuver, a lack of understanding of the "Basic Aerodynamics" of the maneuver will enter you into the NTSB book of fatal statistics. When it comes to the turnback maneuver, it is important to remember the adage: "The Devil is in the Details!".

### 7.0 References

1. Lowry, J.T., "Performance of Light Aircraft, AIAA Publications, 1999

## Appendix A: A Conservative Approach to Account for the Effect of the Wind on the Turnback Maneuver

The presence of horizontal winds during the turnback maneuver clearly affects the outcome of the turnback maneuver. Although the analysis described in the paper does not consider the wind, one can develop a relatively simple conservative correction to get a handle of the effect of the wind. Figures A1 and A2 show the effect of a horizontal wind on the flight path angle. In this example we consider just the headwind/tailwind cases. If the flight experiment is performed at a towered airport, the current winds from the Tower can be used for the wind speed and direction. In the case of a headwind, the equation in Figure A2 can be used to obtain the climb angle relative to the air mass, i.e. $\gamma$, given the climb angle relative to the ground, i.e. $\gamma_{G}$


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Figure A1: Effect of a Horizontal Wind on the Glide Path Angle

## How Does Wind Affect Turn-back Maneuver?

- Relationship exists between the climb/glide angle of the aircraft relative to the air mass and relative to that of the earth

$$
\mathrm{V}_{T} \operatorname{Sin} \gamma=\mathrm{V}_{G} \operatorname{Sin} \gamma_{G} \circlearrowright \begin{aligned}
& \text { Rate of Descent } \\
& \text { Does Not Change }
\end{aligned}
$$

$$
\begin{aligned}
& \text { where } V_{T}=\text { True airspeed } \\
& V_{G}=\text { Ground speed (shallow flight path approx.) } \\
& Y=\text { Climb/glide path angle relative to air mass } \\
& Y_{G}=\text { Climb/glide path angle relative to earth }
\end{aligned}
$$



Figure A2: Effect of a Horizontal Wind on the Climb/Glide Path Angle

The above corrections can also be utilized when the wind is blowing from an arbitrary direction. In the case of a crosswind, the direction of the turn should be into the wind. Application of the wind triangle results in the ground speed and wind correction angle given by

$$
\begin{align*}
& V_{G}=V_{T} \operatorname{Cos} \sigma-V_{\text {Wind }} \operatorname{Cos} \alpha \\
& S \text { in } \sigma=\left(\frac{V_{\text {Wind }}}{V_{T}}\right) S \text { in } \alpha \tag{44}
\end{align*}
$$

Here $\alpha$ is the angle of the wind relative to the runway heading during the climb, and relative to the inbound course being flown directly to the DER in Segment 2. As a conservative estimate, one can use the intercept angle in the no-wind scenario as the inbound course to the DER. The variable $\sigma$ is the wind correction angle during both the climb-out, and the wings-level glide while inbound to the DER on Segment 2. Note that the glide path angle in Segment 2 will be a function of $\frac{D}{R_{1}}$ (since the intercept angle is not constant). Thus, for a given windspeed and wind direction, the resultant glide path angle as a function of distance from the DER can be determined. However, in the climb phase, the value of both $\sigma$ and ${ }_{\alpha}$ would be constant. Note that the key parameters
that control the wind correction angle are the windspeed ratio $\frac{V_{\text {Wind }}}{V_{T}}$, and the relative wind angle $\alpha$.

Using the determined climb and glide angles relative to the ground, one can employ the analysis described in the paper to obtain a first estimate of the effect of the wind on the height above the DER, and hence the RMRL. Figure A3 shows the effect of a 15-knot headwind on the resultant RMRL. One observes a considerable reduction in the required RMRL. However, under strong headwind conditions, there can be a significant consequence of this reduced RMRL when initiating a turnback at large distances from the DER. This is due to the excess altitude that the aircraft may have accumulated during the climb phase and occurs when there is a significant difference between the climb and glide angles. Under this scenario, the aircraft may be too high to land downwind, and thus the only alternative would be to fly downwind and attempt a gliding 180-degree turn to land upwind. If the aircraft does not have enough altitude to initiate the turn at the beginning of the departing runway, the turn may need to occur midfield, which then brings in the issue of obstacles which can be encountered during the final turn. The advantage of the aerodynamic model is that this is issue can be flagged during the preflight briefing while on the ground, which can then alert the Pilot to such a potential situation.

Regarding the estimate of the time beyond the DER before initiating the turnback maneuver, one can use the following equation to estimate ttbi, in the presence of a wind

$$
\begin{equation*}
t_{t b i}=\frac{\left(\frac{\rho_{s l}}{\rho}\right)\left(V_{1}\right)^{2}}{9.53 V_{G}} \tag{45}
\end{equation*}
$$

In the case of sea level departure with a 15-knot headwind and a 70 KCAS climb speed, the delay time before initiating a turnback would be about 10 seconds after passing over the DER.


Figure A3: Effect of 15-Knot Headwind on the RMRL (C-172, Gross Weight, Sea Level)

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